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$$3\mu^2a^2 = [\mu a + Q\sin(\phi - \theta)]^2 + [\mu b + Q\sin(\theta + \frac{1}{2}\beta - \phi)]^2 \\ + 2[\mu a + Q\sin(\phi - \theta)][\mu b + Q\sin(\theta + \frac{1}{2}\beta - \phi)]\cos\frac{1}{2}\beta \dots (4).$$

Eliminating ϕ between (3) and (4) gives an equation to determine θ .

NUMBER THEORY AND DIOPHANTINE ANALYSIS.

152. Proposed by H. S. VANDIVER, Bala, Pa.

When p is a prime of the form $5n+1$ then there is a positive integer a such that $a^2 \equiv 5 \pmod{p}$. Show that $\left(\frac{a \pm 1}{p}\right) = \pm \left(\frac{-2a}{p}\right)$, according as p is of the form $5n+1$ or $5n-1$.

Solution by G. B. M. ZERR, A. M., Ph. D., Philadelphia, Pa.

A general expression involving all cases is, apparently, not easily deduced. The following cases hold.

(1) Let $n=2$. Then $p=5n+1=11$. $a=4$, $a=p-4=7$, $a=2p-7=15$, $a=3p-15=18$, etc.

(2) Let $n=4$. Then $p=5n-1=19$, $a=9$, $a=p-9=10$, $a=2p-10=28$, $a=3p-28=29$, etc.

(3) Let $n=6$. Then $p=5n+1=31$, $a=6$, $a=p-6=25$, $a=2p-25=37$, $a=3p-37=56$, etc.

(4) Let $n=8$. Then $p=5n+1=41$, $a=13$, $a=p-13=28$, $a=2p-28=54$, $a=3p-54=69$, etc.

(5) Let $n=12$. Then (b) $p=5n+1=61$; (c) $p=5n-1=59$.

(b) $a=26$, $a=p-26=35$, $a=2p-35=87$, $a=3p-87=96$, etc.

(c) $a=8$, $a=p-8=51$, $a=2p-51=67$, $a=3p-67=110$, etc.

For every value of n that makes $5n \pm 1$ a prime, we can find values for a satisfying the condition. It is also easy to see that a can have an infinity of values for each case.

In (1), (3), (4), (5) (b), $(a+1)^{\frac{1}{2}(p-1)} = (-2a)^{\frac{1}{2}(p-1)} \equiv -1 \pmod{p}$.

In (2), (5) (c), $(a-1)^{\frac{1}{2}(p-1)} = -(-2a)^{\frac{1}{2}(p-1)} \equiv 1 \pmod{p}$.

$\left(\frac{a \pm 1}{p}\right) = \pm \left(\frac{-2a}{p}\right)$, in the cases examined, according as $p=5n \pm 1$.

A general solution is desired. ED. F.

152. Proposed by H. S. VANDIVER, Bala, Pa.

Prove geometrically:

$\sum_{n=1}^{\frac{1}{2}(p-1)} \left[\frac{n^2}{p} \right] = \frac{p-3}{4} \cdot \frac{p-1}{2} - \sum_{n=1}^{\frac{1}{2}(p-4)} \left[\sqrt{np} \right]$, where $p \equiv 3 \pmod{4}$ and $\left[\frac{k}{p} \right]$ represents the greatest integer in k/p .

Remarks by G. B. M. ZERR, A. M., Ph. D., Philadelphia, Pa.

In the second term of the second member of the equation, $\frac{1}{2}(p-4)$ should be $\frac{1}{2}(p-3)$. The equation is generally but not universally true as is shown by induction in what follows.

Let $p=4m+3$, then $\frac{1}{2}(p-1)=2m+1$, $\frac{1}{2}(p-3)=m$,

$$\sum_{n=1}^{\frac{1}{2}(p-1)} \left[\frac{n^2}{p} \right] = \frac{p-3}{4} \cdot \frac{p-1}{2} - \sum_{n=1}^{\frac{1}{4}(p-3)} [\sqrt{(np)}],$$

or as follows:

$$\begin{array}{ll} A = B - C, \\ m=2, & 3 = 10 - 7, \\ m=3, & 7 = 21 - 14, \\ m=4, & 11 = 36 - 25, \\ m=5, & 18 = 55 - 37, \\ m=7, & 34 = 105 - 71. \end{array}$$

If one of the $[n^2/p]$ is an exact quotient, and hence one of the $[\sqrt{(np)}]$ rational, the equation is $A=1+B-C$.

$$\begin{array}{l} m=6, \quad p=27, \quad [9^2/p]=3, \quad \sqrt{(3 \times 27)}=9, \\ m=15, \quad p=63, \quad 21^2/p=7, \quad \sqrt{(7 \times 63)}=21. \end{array}$$

$$\therefore A=1+B-C, \quad m=6 \dots 25=1+78-54, \quad m=15 \dots 153=1+465-313.$$

If two of the $[n^2/p]$ are exact quotients, and hence two of the $[\sqrt{(np)}]$ rational, the equation becomes $A=2+B-C$.

$$m=18, \quad p=75, \quad 15^2/p=3, \quad \sqrt{(3 \times p)}=15, \quad 30^2/p=12, \quad \sqrt{(12 \times p)}=30.$$

$\therefore A=2+B-C$ becomes $219=2+666-449$ for $m=18$. $A=t+B-C$ is the true universal equation.

The geometric proof in this solution is wanting. Who can produce it? Ed. F.

155. Proposed by PROF. R. D. CARMICHAEL, Anniston, Alabama.

If p and q are primes and m and n are any integers, find the cases in which the equation $p^m - q^n = 1$ may be satisfied.

Remarks by G. B. M. ZERR, A. M., Ph. D., Philadelphia, Pa.

Some values, found by inspection, are given in the following table: